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STRUCTURAL EFFICIENCY OF ALUMINUM  
MULTIWEB BEAMS AND Z-STIFFENED  
PANELS REINFORCED WITH  
FILAMENTARY BORON-EPOXY COMPOSITE

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FILAMENTARY BORON-EPOXY COMPOSITE

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SUMMARY

The gain in structural efficiency achieved by reinforcing the aluminum cover skins of multiweb beams and Z-stiffened panels with rectangular boron-epoxy stiffeners located midway between conventional webs or stiffeners is calculated. The effectiveness of adding boron-epoxy reinforcement to the outstanding flange of the Z-stiffeners in Z-stiffened panels is also evaluated. Cover reinforcement reduced the mass of the multiweb beams by about 25 percent and reduced the mass of Z-stiffened panels by about 15 percent. The flange reinforcement reduced the mass of the Z-stiffened panels by another 8 percent at low values of the loading index but had little effect on mass at higher values of the loading index.

Attainable efficiency was limited by failure modes characteristic of the boron-epoxy stiffening used. Multiweb beam efficiency was limited by a buckling mode entailing twisting of the composite stiffener and cover skin about the stiffener-skin juncture. Buckling occurred at a lower stress level than might have been anticipated from experience gained from all-metal structures because this mode is closely associated with twisting stiffness of the stiffener which is small for composite stiffeners. Efficiency of the Z-stiffened panels was limited in a similar manner but was further limited by two modes of failure involving buckling of the outstanding flange of the stiffeners when reinforcement was added to the outstanding flange. One mode entailed twisting of the flange and web of the stiffener about the flange-web juncture, but the principal limiting mode entailed buckling of the reinforced flange as a "beam column" with deflection normal to the web of the Z-stiffener. The susceptibility of Z-stiffeners to this mode of failure suggests that different shape stiffeners should probably be used in applications where the stiffener is reinforced with an advanced composite.

## INTRODUCTION

Considerable interest has recently been given to reinforcing metal aircraft structures with advanced composite materials in order to obtain stiffer and stronger structures. (See refs. 1 and 2.) The low density and high strength and stiffness of boron or graphite reinforcements make such a scheme attractive. However, many problem areas inherent in the efficient application of advanced composites to metal structures must be solved before such structures are feasible. One of the problems is concerned with the effective distribution of the advanced reinforcements for maximum enhancement of the strength and/or stiffness characteristics of the metal structures. The present study is concerned with one phase of this broad problem.

A study is made of the gains in strength obtained by reinforcing the compression cover skin of conventional aluminum wing structures with rectangular stringers consisting of unidirectional boron fibers embedded in an epoxy matrix. Concepts are investigated (fig. 1) in which the composite stiffener is located midway between the more conventional stiffeners which reinforce compression covers, that is, between the webs of a multiweb beam or between the stringers of a Z-stiffened panel typical of those used for compression covers of wing structures. The addition of composite stiffening to the outstanding flange of the Z-stiffener is also studied. These configurations were chosen for study after preliminary studies indicated that "lumping" the unidirectional boron-epoxy composite into a central rectangular stiffener was more effective in preventing buckling than the alternate scheme of "smearing" the composite over the entire plate. Moreover, the preliminary study indicated that a central stiffener was as effective as two or more stiffeners. The stiffeners in multiple stiffening arrangements were deep and narrow and hence more susceptible to buckling failures involving twisting of the stiffeners about the plate-stiffener juncture.

The study is conducted along conventional lines with the use of parametric studies and structural efficiency plots for determining structures with a low mass and high strength. The buckling behavior of various elements which make up the structures depicted in figure 1 is discussed; then results are given of calculations to determine the structural efficiency of multiweb beams and Z-stiffened panels.

## SYMBOLS

The units for physical quantities in this paper are given both in the U.S. Customary Units and in the International System of Units (SI). Factors relating the two systems are given in reference 3; those factors used in the present paper are given in appendix A.

$A_i$	mass of compression cover and webs of multiweb beam per unit width of beam expressed as an area of aluminum of same mass
$A_e$	area of edge member
$A_s$	area of composite stiffener
$b$	width of reinforced plate; distance between Z-stiffeners in Z-stiffened panel; distance between webs in multiweb beam
$b_F$	width of flange of Z-stiffener
$b_w$	depth of web of Z-stiffener
$D$	bending stiffness of plate, $\frac{Et^3}{12(1 - \mu^2)}$
$D_w$	bending stiffness of web of Z-stiffener
$d = \frac{b}{2}$	
$E$	Young's modulus of plate material
$E_e$	Young's modulus of material in edge stiffener
$E_s$	Young's modulus of composite stiffener material in direction of stiffener
$h$	depth of multiweb beam
$k_w = \frac{N_w b_w^2}{\pi^2 D_w}$	
$l$	length of Z-stiffened panel
$M_i$	bending moment on multiweb beam per unit width of beam
$m$	depth of rectangular composite stiffener measured in thicknesses of plate being reinforced

N	compressive load per unit of width on plate reinforced with a central rectangular composite stiffener or on Z-stiffened panel constructed from such plates (includes load on plates and all stiffening members except edge members)
$N_p$	that portion of compressive load per unit of width of plate that is carried by plate
$N_w$	compressive load in web of Z-stiffener per unit of width of web
$N_{eq}$	compressive load per unit of width of plate on metal plate of same mass as composite reinforced plate
n	width of rectangular composite stiffener measured in thicknesses of plate being reinforced
S	plate stiffness, used with subscripts, superscripts, and overhead symbols to denote particular stiffness
$S_Z^V$	stiffness of Z-stiffener with reinforced flange
t	thickness of plate
$t_F$	thickness of reinforced flange of Z-stiffener
$t_w$	thickness of web of Z-stiffener
$\bar{t}$	mass of reinforced plate or Z-stiffened panel per unit of width expressed as a thickness of aluminum of same mass
w	deflection normal to plate
x,y	coordinates along and across plate, respectively
$\eta$	ratio of area of composite used to reinforce the outstanding flange of Z-stiffener to that used to reinforce plate
$\lambda$	buckle length
$\mu$	Poisson's ratio

## BUCKLING OF PLATES AND COMPONENTS

Calculations for structural efficiency require determining the load at which structures fail in the various possible failing (buckling) modes. Interaction between buckling modes is normally neglected and is neglected in many calculations of the present study. The stiffened plate between webs and Z-stiffeners (fig. 1) is assumed to buckle into one of two modes with no interaction between the modes. The modes are illustrated in figure 2. One mode entails lateral deformations, symmetric about the composite stiffener, of both the plate and stiffener; this mode predominates for plates with small stiffeners. The other involves asymmetric lateral deformations of the plate and a twisting of the plate and composite stiffener about the plate-stiffener juncture. The latter mode predominates in plates with heavy stiffeners.

Local buckling of Z-stiffeners is also presumed to occur in either of two modes. One mode entails relatively short buckles and consists of twisting of the outstanding flange and web of the stiffeners about the flange-web juncture. This mode is called the "local buckling mode" in figure 3(a). The other mode is illustrated by figure 3(c) and consists of a coupling of the local mode (fig. 3(a)) and the so-called "twisting mode" (fig. 3(b)). Calculations for this latter and more complex mode were made only for buckling into relatively long wavelengths. The "coupled mode" entails lateral deformations of the flange in a "beam-column" action and might appropriately be called the "beam-column mode." The local buckling mode (fig. 3(a)) normally governed for stiffeners with little or no flange reinforcement; the coupled mode governed for stiffeners with considerable flange reinforcement.

Calculations for buckling modes are discussed in this section of the paper. For the calculations where the reinforcement is treated as a beam and where the only important property of the reinforcement is Young's modulus in the direction of the fibers, a value equal to three times that of aluminum or 31 500 ksi ( $217 \text{ GN/m}^2$ ) is used. For other calculations where the reinforcement is treated as a plate or a layer of a plate and where all moduli are needed, values are used which were obtained with the use of reference 4 for a composite with equal parts by volume of boron and epoxy; values of modulus and Poisson's ratio of the boron and epoxy of 60 000 ksi ( $414 \text{ GN/m}^2$ ) and 0.30 and 500 ksi ( $3.45 \text{ GN/m}^2$ ) and 0.40, respectively, were used in making the calculation for moduli. In all calculations the density of the boron-epoxy composite was taken to be 65 percent of that of aluminum. These material properties were chosen from the diverse values published in the literature. Some of the more recent studies indicate that the values used herein for shear modulus and density of the boron composite are somewhat low. The use of a higher value for shear modulus would have resulted in somewhat more efficient structures; the use of a greater density, in somewhat less efficient structures.

## Symmetric Buckling of a Plate with a Central Rectangular Composite Stiffener

The buckling studies of references 5 to 8 were used to calculate the strength of plates with light stiffening. References 5 and 6 treat plates symmetrically stiffened about the midplane of the plate; references 7 and 8 extend the symmetrically stiffened plate studies to plates of interest here with eccentric (one-sided) stiffening.

Results of calculations for buckling of a long simply supported plate stiffened with a central composite longitudinal stiffener are given in figure 4. The ordinate of figure 4 is the load carried by the stiffened plate, including the load carried by the central stiffener, divided by the load carried by an unstiffened plate of the same mass. The loads in both instances are buckling loads resulting from uniform axial compression.

Two sets of curves are presented in figure 4, one for plates without edge stiffening ( $\frac{E_e A_e}{E_{bt}} = 0$ ) such as shown by the inset diagram and one for plates with an edge stiffener ( $\frac{E_e A_e}{E_{bt}} = 0.5$ ) attached along each unloaded edge in addition to the central composite stiffener. The load  $N$  does not include the load carried by the edge members. It is seen that the effect of edge stiffening is not great; and the efficiency calculations given later for multiweb beams and Z-stiffened panels conservatively neglect the small stabilizing effect of edge stiffeners.

The advantage shown in figure 4 for stiffened plates at low values of stiffening ratio is generally not available in configurations of practical interest. For example, the composite stiffener resulting from the use of the figure for large values of  $m$  and small values of stiffening ratio is deep and narrow; and the structure would buckle in the asymmetric mode rather than in the symmetric mode exemplified in the figure. The asymmetric mode is considered in the next section of the paper.

## Asymmetric Buckling of a Plate with a Central Rectangular Composite Stiffener

Buckling of a plate with a central stiffener in the asymmetric model entails twisting of the plate and composite stiffener about the plate-stiffener juncture. Procedures for estimating the onset of such buckling in structural components are highly refined. (See, for example, refs. 9 and 10.) All that is needed are tables, charts, or expressions for the stiffness of plate elements that make up the component; buckling is determined from the condition that the sum of the stiffnesses of the various elements at a joint (intersection of elements) vanishes at buckling. The stiffness of the metal plate elements can be obtained

from reference 11 or 12; the stiffness of the composite stiffener is given by equation (B1) of appendix B which was adapted from a similar equation in reference 13.

The buckling equation for a simply supported plate with a central rectangular stiffener is given by

$$2\hat{S}^{II} + \tilde{S}^{III} = 0 \quad (1)$$

where the superscripts on the stiffnesses are the notation of reference 11 and the caret and tilde over the symbols denote properties associated with the isotropic metal plate and the orthotropic composite stiffener, respectively.

Typical results of buckling calculations made with the use of stability equation (1) are presented in figure 5 by the solid curve. The dashed curves denote buckling in the symmetric mode for plate-thickness ratios of interest in the present study. As mentioned earlier, plates with deep stiffeners buckle in the asymmetric mode whereas those with shallow stiffeners buckle in the symmetric mode. The stiffener at which transition from symmetric buckling to asymmetric buckling occurs is shallower than one might expect from experience with all-metal structures. Metal stiffeners have a higher shear stiffness than the composite stiffeners considered here and buckling calculations for the asymmetric mode depend heavily upon the shearing stiffness of the stiffener because the deformation of the stiffener in the asymmetric mode is largely one of twisting. Calculations such as those shown in figure 5 were used to determine the buckling strength of the plate-stiffener combinations considered in the structural efficiency calculations presented later.

#### Local Buckling of a Z-Stiffener by Twisting of Outstanding Flange

##### About Flange-Web Juncture

The buckling equation for a Z-stiffener assumed to be simply supported at the juncture between the attachment flange and the web of the stiffener is given by

$$\hat{S}^{II} + \tilde{S}^{III} = 0 \quad (2)$$

where the caret and tilde are used to denote properties associated with the isotropic web and the orthotropic reinforced flange of the Z-stiffener, respectively; the Roman numeral superscripts are the notation of reference 11. The stiffness  $\tilde{S}^{III}$  is given by equation (B1); values of the stiffness  $\hat{S}^{II}$  can be obtained from reference 11.

Results of calculations made with the use of equation (2) are shown in figure 6 for a Z-stiffener with an area of reinforcement of the outstanding flange equal to two times the

area of the flange ( $\frac{t_F}{t_W} = 3.0$ ). This ratio is the largest reinforcement ratio considered in panel geometries for the efficiency study given later.

The curve for buckling of a Z-stiffener without reinforcement (fig. 6) indicates buckling of a web which is, for practical purposes, simply supported by the outstanding flange; that is, the buckling coefficient and buckle length are approximately those of a plate simply supported along the unloaded edges. On the other hand, the curve for buckling of the stiffener with flange reinforcement has two minimums in the range of buckling coefficients of interest, one associated with buckling of the web restrained by the flange and the other associated with buckling of the flange restrained by the web. Buckling is given by the lower minimum. Calculations such as those exemplified in figure 6 were used to insure the use of panel geometries in the efficiency studies of the present investigation which do not fail by twisting of the outstanding flange of the Z-stiffener about the flange-web juncture.

#### Local Buckling of a Z-Stiffener by Deflection of Outstanding Flange of Stiffener Normal to Web of Stiffener

Methods for predicting the buckling strength of stiffeners whose flanges have insufficient stiffness as a beam column to support the web of the stiffener under load are not as refined as those for predicting modes entailing a rotation of the flange and web elements about the flange-web juncture. Consequently, contemporary practice relies on empirical criteria such as the one given by equation (14) of reference 14 for insuring that buckling in the "beam-column" mode will not predominate. Empirical criteria are not available for the reinforced flanges considered in the present study, however, and an analytical criterion had to be used.

The analysis of reference 15 of the web and flange of a Z-stiffener as an integral unit is very suitable for the present study. This analysis is readily extended to stiffeners with composite-reinforced flanges and incorporates much of the structural behavior known to be important, behavior which is neglected by simpler analyses. For example, the analysis of reference 15 includes, in an approximate but presumably satisfactory manner, (a) the continuity of strain and rotation between the web and flange of the stiffener, (b) the restraint provided to the web of the stiffener by the face sheet of the skin-stringer panel, and (c) the possibility of a twisting failure of the flange as a beam. All these effects are neglected in simpler analyses. (See, for instance, ref. 16.)

The buckling criterion developed by extending the study of reference 15 is given in appendix C; results of buckling calculations made with the use of the criterion are presented in figure 7. The curve of figure 7 for no flange reinforcement ( $\frac{t_F}{t_W} = 1.0$ ) indicates that such a flange provides the equivalent of simple support ( $k_W = 4.0$ ) to webs with  $\frac{b_W}{t_W}$

greater than about 27 but that reinforced flanges provide "simple support" only to much deeper webs; buckling of most of the panels with reinforced flanges considered in the efficiency study given later was governed by the mode of buckling considered in figure 7.

## STRUCTURAL EFFICIENCY OF PLATES AND COMPONENTS

The results of structural efficiency calculations based on the buckling calculations discussed in the previous section are presented in this section of the paper. The structural efficiency of a plate reinforced with a central composite stiffener is discussed; then calculations for multiweb beams and Z-stiffened panels of which the plates are an integral part are discussed.

### Plates

Figure 8 shows the buckling efficiency of reinforced plates in relation to the buckling and crippling efficiency of plates without reinforcement and to hypothetical unstiffened plates which are not susceptible to buckling and which are stressed to 72 ksi (496 MN/m<sup>2</sup>), the yield stress of 7075-T6 aluminum alloy. The curve for buckling of plates without reinforcement is based on the assumption of simply supported unloaded edges, and the curve for crippling was computed with the use of reference 14. The composite-reinforced plate is clearly superior to unreinforced plates particularly at the higher loading indices. Moreover, the reinforced plate is stiffer than the conventional plate; the reinforced plate is free of buckles and has a working stress in the plate material less than the proportional limit stress of the plate material, 7075-T6 aluminum alloy.

The curve shown for reinforced plates in figure 8 is the result of calculations for plates reinforced with a stiffener having a depth of  $8t$  and a width of  $2t$ . The use of this stiffener is a result of systematic buckling calculations in which stiffener depth was varied in increments of  $t$  and stiffener width in increments of  $\frac{1}{2}t$ . The  $2t$  dimension is the smallest considered in the present study on the premise that this width is probably required in order to achieve a good bond between the stiffener and plate. The dimension  $8t$  was selected because calculations with progressively greater values than  $8t$  indicated little or no improvement in efficiency; and the reinforcement ratio  $\left(\frac{E_S A_S}{E_{bt}}\right)$  associated with the  $m = 8, n = 2$  stiffener is in the range of successful application with all-metal structures. Calculations for larger values of  $m$  and  $n$  which indicate equally efficient structures must be viewed with less confidence as a result of the large reinforcement ratios associated with the structures. The principal advantage of plates with stiffeners having larger values of  $m$  and  $n$  is that consideration of such structures would extend the curve of figure 8 further to the right without apparent change in course.

### Multiweb Beams

The buckling calculations of the previous section of the paper are readily incorporated into structural efficiency calculations for a multiweb beam with a compression cover reinforced by composite stiffeners between each web. The results of such calculations are shown in figure 9 and compared with similar calculations for conventional multiweb beams which fail by buckling or by crippling. A curve for a hypothetical beam without webs but with a compression cover that sustains the yield stress without failing is also shown. The curves of figure 9 were obtained by calculations similar to those used in reference 17 except a linear bending stress distribution for the beam webs as proposed in reference 18 was used instead of the distribution used in reference 17.

Figure 9 indicates that reinforcement of the compression cover results in a substantial improvement in efficiency at all loading indices considered if one compares buckling results of reinforced beams with buckling results of conventional beams. The improvement is limited to the higher loading indices if one compares the buckling results of reinforced beams with the crippling results of conventional beams.

In figure 9, as in figure 8, the particular curve shown for reinforced beams was computed with the use of a stiffener with  $m = 8$  and  $n = 2$  because systematic calculations indicated that the  $m = 8, n = 2$  stiffener was superior to stiffeners of other shapes. The curve can be extended to the right without significant change in course by considering stiffeners with larger values of  $m$ . However, as discussed for plates, stiffeners with values of  $m$  substantially greater than 8 are beyond the range of experience gained with all-metal construction and confidence in results obtained using such stiffeners is not as great as that for the smaller stiffeners.

### Z-Stiffened Panels

Two configurations of reinforced Z-stiffened panels are considered in this section of the paper, one with reinforcement only on the cover skin of the panel and one with reinforcement on both the cover skin and the outstanding flange of the Z-stiffener. The basic Z-stiffener used in both studies is a section of constant thickness in which the outstanding flange width is 0.40 of the web depth and in which the attachment flange has a flat portion for attachment to the cover skin that has a width of 7.5 times the thickness of the section. The flanges and webs are joined by circular arcs with an inside bend radius equal to the thickness of the section. Although this Z-section may not be optimum, it is representative of those that have evolved from strength and efficiency studies of unreinforced panels and one which is adequate for demonstrating the potential gains that can be expected from the addition of composite reinforcement.

Panels with cover-skin reinforcement only. - The structural efficiency calculations for Z-stiffened panels with cover-skin reinforcement were conducted along the lines of those given in reference 19 for unreinforced panels; that is, the capacity of the panel for carrying load without buckling of either the composite-reinforced cover skin or the Z-stiffeners was taken as the strength criterion. Both the plate and the Z-stiffeners were assumed to be simply supported at the junctures between the plate and Z-stiffeners in computing buckling load.

Results of calculations to determine the efficiency of Z-stiffened panels with boron-reinforced cover skins are given in figure 10. The addition of boron reinforcement results in about a 15-percent lighter structure at all values of the loading index. The reinforced panels at high values of the index are lighter than all-metal panels working at the yield stress.

Here, as in the studies of plates and multiweb beams, a composite stiffener of  $2t$  width and  $8t$  depth is determined as the most desirable configuration. This result is not completely unexpected. Interaction between the reinforced cover skins and webs of the multiweb beams or the stiffeners of the Z-stiffened panels is neglected in the efficiency studies and it is not surprising that the configurations resulting from the plate study are desirable configurations when used as the cover skin of multiweb beams or Z-stiffened panels. As noted previously for plates and multiweb beams, the curve for Z-stiffened panels with reinforced cover skins (fig. 10) can also be extended to the right by considering composite stiffeners with larger values of  $m$  and  $n$ .

Panels with reinforcement on both the cover skin and outstanding flanges. - Results of calculations for panels with boron reinforcement on both the cover skin and on the outstanding flanges of the Z-stiffeners (fig. 1(c)) are given in figure 11 and compared with those for panels with reinforcement only on the cover skin. The addition of reinforcement to the outstanding flanges results in about an 8-percent lighter structure at low values of the loading index but in progressively smaller savings at the higher indices. The particular curve shown in figure 11 was calculated for a stiffener with  $m = 8$  and  $n = 2$  and for a ratio of flange reinforcement to plate reinforcement  $\eta$  of 0.50. The result given in figure 11 depends heavily upon the criteria for buckling discussed for calculating buckling of Z-stiffeners with reinforced outstanding flanges.

The reinforcement ratio  $\eta$  of 0.50 was chosen after systematic calculations with other values indicated that little improvement in efficiency was obtained by using values greater than 0.50. Figure 12 illustrates the effect of flange reinforcement on structural efficiency. The ordinate of the figure is a measure of efficiency because the parameter  $\bar{t}/l$  is nearly proportional to  $\sqrt{N/l}$  for panels designed to achieve low mass and high strength. Figure 12 indicates that although small amounts of flange reinforcement enhance structural efficiency, large amounts result in reduced efficiency.

The use of Z-stiffeners with their outstanding flanges reinforced with a boron-epoxy composite does not appear to result in a very proficient structure and other types of reinforced stiffeners should be investigated. For instance, a reinforced hat-section stiffener may perform better than the Z-stiffeners. The hat-section stiffener would not appear to be as susceptible to failure in the beam-column mode which limits the capacity of Z-stiffeners for carrying load. Such studies are however beyond the scope of the present study.

#### CONCLUDING REMARKS

Results of calculations to determine the structural efficiency achieved by reinforcing the compression cover of conventional multiweb beams and Z-stiffened panels with boron-epoxy stiffeners located midway between the conventional webs and stiffeners are discussed. In addition, calculations in which the outstanding flanges of the Z-stiffeners in Z-stiffened panel construction were also reinforced are given. The reinforced structures were found to be considerably more efficient than structures without reinforcing.

Cover-skin reinforcement was more effective in reducing the mass of multiweb beams than of Z-stiffened panels. The mass of the reinforced multiweb beams considered was typically 75 percent of that of beams without reinforcement whereas the mass of reinforced Z-stiffened panels was typically 85 percent of that of panels without reinforcement. The addition of reinforcement to the outstanding flanges of the Z-stiffeners in the Z-stiffened panel construction reduced structural mass by another 8 percent at low values of the loading index but had little effect on mass at high values of the loading index.

Attainable efficiency was limited by failure modes characteristic of the boron-epoxy stiffening used. Multiweb beam efficiency was limited by a buckling mode entailing twisting of the composite rectangular stiffener and metal cover skin about the stiffener-skin juncture. Buckling occurred at a lower stress level than might have been anticipated from experience gained from all-metal structures because this mode is closely associated with twisting stiffness of the stiffener which is small for composite stiffeners. Efficiency of the Z-stiffened panels was limited in a similar manner but was further limited by two modes of failure involving buckling of the outstanding flange of the stiffeners when reinforcement was added to the outstanding flange. One mode entailed twisting of the flange and web of the stiffener about the flange-web juncture, but the principal limiting mode entailed buckling of the reinforced flange as a beam column with deflections normal to the web of the Z-stiffener. The susceptibility of Z-stiffeners to this mode of failure suggests that different shape stiffeners should probably be used in applications where the stiffener is reinforced with an advanced composite.

Langley Research Center,

National Aeronautics and Space Administration,

Hampton, Va., March 26, 1970.

## APPENDIX A

### CONVERSION OF U.S. CUSTOMARY UNITS TO SI UNITS

The International System of Units (SI) was adopted by the Eleventh General Conference on Weights and Measures, Paris, October 1960 (see ref. 3). Conversion factors for the units used in this report are given in the following table:

Physical quantity	U.S. customary unit	Conversion factor (*)	SI unit (**)
Stress, modulus	ksi	$6.895 \times 10^6$	newtons/meter <sup>2</sup> (N/m <sup>2</sup> )

\*Multiply value given in U.S. customary units by conversion factor to obtain equivalent value in SI units.

\*\*Prefixes to indicate multiple of units are as follows:

Prefix	Multiple
giga (G)	$10^9$
mega (M)	$10^6$

## APPENDIX B

### STIFFNESS OF AN ORTHOTROPIC FLANGE

In the calculations given in the body of the paper for buckling of a plate with a central rectangular composite stiffener in the asymmetric mode and in the calculations for buckling of a Z-section stiffener with a reinforced flange in a mode entailing twisting of the flange and web of the stiffener about the flange-web juncture, the central stiffener and the reinforced flange are treated as orthotropic plate elements. In such calculations the stiffness of the composite plate must be known. The stiffness is given by

$$\frac{4\tilde{S}_{b}^{III}}{\tilde{D}_{22}} = \frac{(\alpha^2 + \beta^2) \left( \beta \frac{A}{B} \sinh \alpha \cos \beta - \alpha \frac{B}{A} \cosh \alpha \sin \beta \right)}{\left( \frac{A}{B} + \frac{B}{A} \right) \alpha \beta \cosh \alpha \cos \beta - \left( \alpha^2 \frac{B}{A} - \beta^2 \frac{A}{B} \right) \sinh \alpha \sin \beta + 2\alpha\beta} \quad (B1)$$

where

$$A = \alpha^2 - \tilde{\mu}_1 \frac{\pi^2 \gamma^2}{\theta_2^2}$$

$$B = \beta^2 + \tilde{\mu}_1 \frac{\pi^2 \gamma^2}{\theta_2^2}$$

$$\alpha = \pi \gamma \sqrt{\theta_1 + \sqrt{\theta_1^2 + \frac{\tilde{k}}{\gamma^2} - 1}}$$

$$\beta = \pi \gamma \sqrt{-\theta_1 + \sqrt{\theta_1^2 + \frac{\tilde{k}}{\gamma^2} - 1}}$$

$$\tilde{k} = \frac{\tilde{N} \tilde{b}^2}{\pi^2 \sqrt{\tilde{D}_{11} \tilde{D}_{22}}}$$

$$\gamma = \frac{\tilde{b}}{\tilde{\lambda}} \theta_2$$

$$\theta_1 = \frac{\tilde{D}_{12}}{\sqrt{\tilde{D}_{11} \tilde{D}_{22}}}$$

## APPENDIX B – Concluded

$$\theta_2 = \sqrt[4]{\tilde{D}_{11}/\tilde{D}_{22}}$$

Equation (B1) is equation (33) of reference 13 corrected for typographical errors. The plate stiffnesses  $\tilde{D}_{11}$ ,  $\tilde{D}_{22}$ , and  $\tilde{D}_{12}$  are those associated with the differential equation of an orthotropic plate in axial compression

$$\tilde{D}_{11} \frac{\partial^4 w}{\partial x^4} + 2\tilde{D}_{12} \frac{\partial^4 w}{\partial x^2 \partial y^2} + \tilde{D}_{22} \frac{\partial^4 w}{\partial y^4} = \tilde{N} \frac{\partial^2 w}{\partial x^2} \quad (B2)$$

and  $\tilde{\mu}_1$  denotes Poisson's ratio associated with bending of the orthotropic plate in the axial or x-direction,  $\tilde{b}$  is the width of the orthotropic plate,  $\tilde{N}$  is the compressive load per unit of width in the plate, and  $\tilde{\lambda}$  is the buckle length of the plate.

## APPENDIX C

### BUCKLING CRITERION FOR PANELS STIFFENED BY Z-SECTION STRINGERS IN WHICH OUTSTANDING FLANGE OF STRINGER IS REINFORCED WITH BORON-EPOXY COMPOSITE

The buckling criterion for a Z-section stiffener simply supported at the juncture between the web of the stiffener and the cover plate to which it is attached is given by

$$\frac{S_Z^V b_W}{D_W} = 0 \quad (C1)$$

where  $S_Z^V$  denotes the stiffness of the Z-stiffener about the juncture. An expression for the stiffness of an all-metal Z-section stiffener is given by equation (8) of reference 15. This equation can be generalized to include Z-section stiffeners with reinforced outstanding flanges of interest in the present study by redefining the parameters  $K$ ,  $L$ ,  $P$ ,  $Q$ ,  $\alpha_1$ , and  $\alpha_2$  of the reference. The generalized parameters are given by

$$K = \frac{\pi E_F I_F}{D_W b_W} \left( \frac{b_W}{\lambda} \right)^3 - \frac{\pi E_F A_F}{E t_W b_W} k_W \frac{b_W}{\lambda}$$

$$L = \frac{\pi G_F J_F}{D_W b_W} \frac{b_W}{\lambda} - \frac{\pi^3 E_F I_P}{E t_W b_W} k_W \frac{b_W}{\lambda}$$

$$P = \sqrt{k_W} - (1 - \mu) \frac{b_W}{\lambda}$$

$$Q = \sqrt{k_W} + (1 - \mu) \frac{b_W}{\lambda}$$

$$\alpha_1^2 = \left( \frac{b_W}{\lambda} \right)^2 + \sqrt{k_W} \frac{b_W}{\lambda}$$

$$\alpha_2^2 = \left( \frac{b_W}{\lambda} \right)^2 - \sqrt{k_W} \frac{b_W}{\lambda}$$

where  $\lambda$  is buckle length,  $E_F$  and  $G_F$  are the extensional and shear moduli of the reinforced flange,  $E$  and  $\mu$  are the modulus and Poisson's ratio of the metallic web and  $A_F$ ,  $I_F$ ,  $J_F$ , and  $I_P$  denote the following respective flange properties: area,

## APPENDIX C – Concluded

moment of inertia about centroid of flange, torsion constant, and polar moment of inertia about intersection of flange and web.

Preliminary buckling calculations made in the present study with the use of equation (C1) indicated that the assumption of simple support was not appropriate for long wavelength buckling and unduly low buckling stresses resulted from its use. In the calculations presented in the body of the report, the criterion

$$\frac{S_Z^V b_W}{D_W} = -1.0 \quad (C2)$$

was used to remove the conservatism inherent in the simple-support assumption.

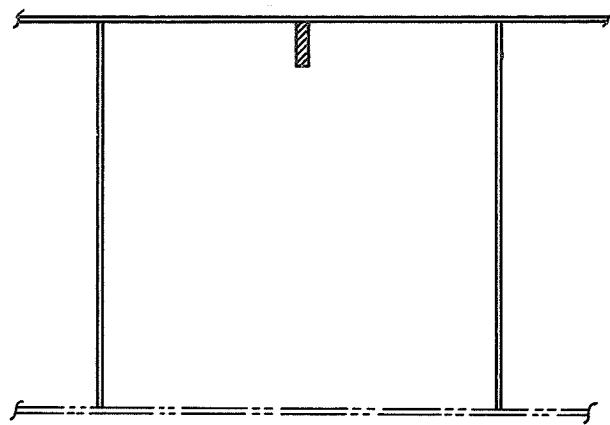
The criterion expressed by equation (C2) is approximate because the magnitude of the stiffness  $\frac{S_Z^V b_W}{D_W}$  at buckling depends to a degree upon stress level and panel geometry.

However, the stiffness  $\frac{S_Z^V b_W}{D_W}$  is quite insensitive to stress level as well as to panel geometry if geometry is limited to configurations of balanced design as in the present study and if the buckling mode of interest is limited to one in which the flange initiates buckling by bending as a beam, the range of interest not covered by calculations for buckling in the "local" mode. (See fig. 3(a).) Hence, although the criterion (eq. (C2)) is convenient and sufficiently accurate for use in the present study, it is not recommended for general application to panels of diverse geometry where individual calculations to determine the magnitude of  $\frac{S_Z^V b_W}{D_W}$  should be made.

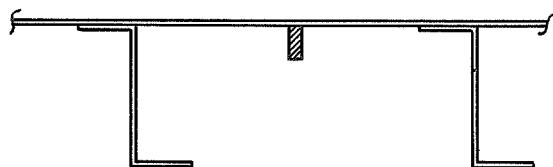
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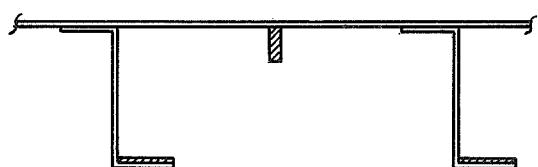
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(a) Multiweb beam.



(b) Z-stiffened panel.



(c) Z-stiffened panel with additional reinforcement on outstanding flange of stiffener.

Figure 1.- Structural configurations studied.



(a) Symmetrical.

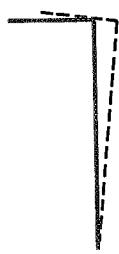


(b) Asymmetrical.

Figure 2.- Buckling modes of plates reinforced with a central rectangular composite stiffener.



(a) Local.



(b) Twisting.



(c) Coupled.

Figure 3.- Buckling modes of Z-stiffeners.

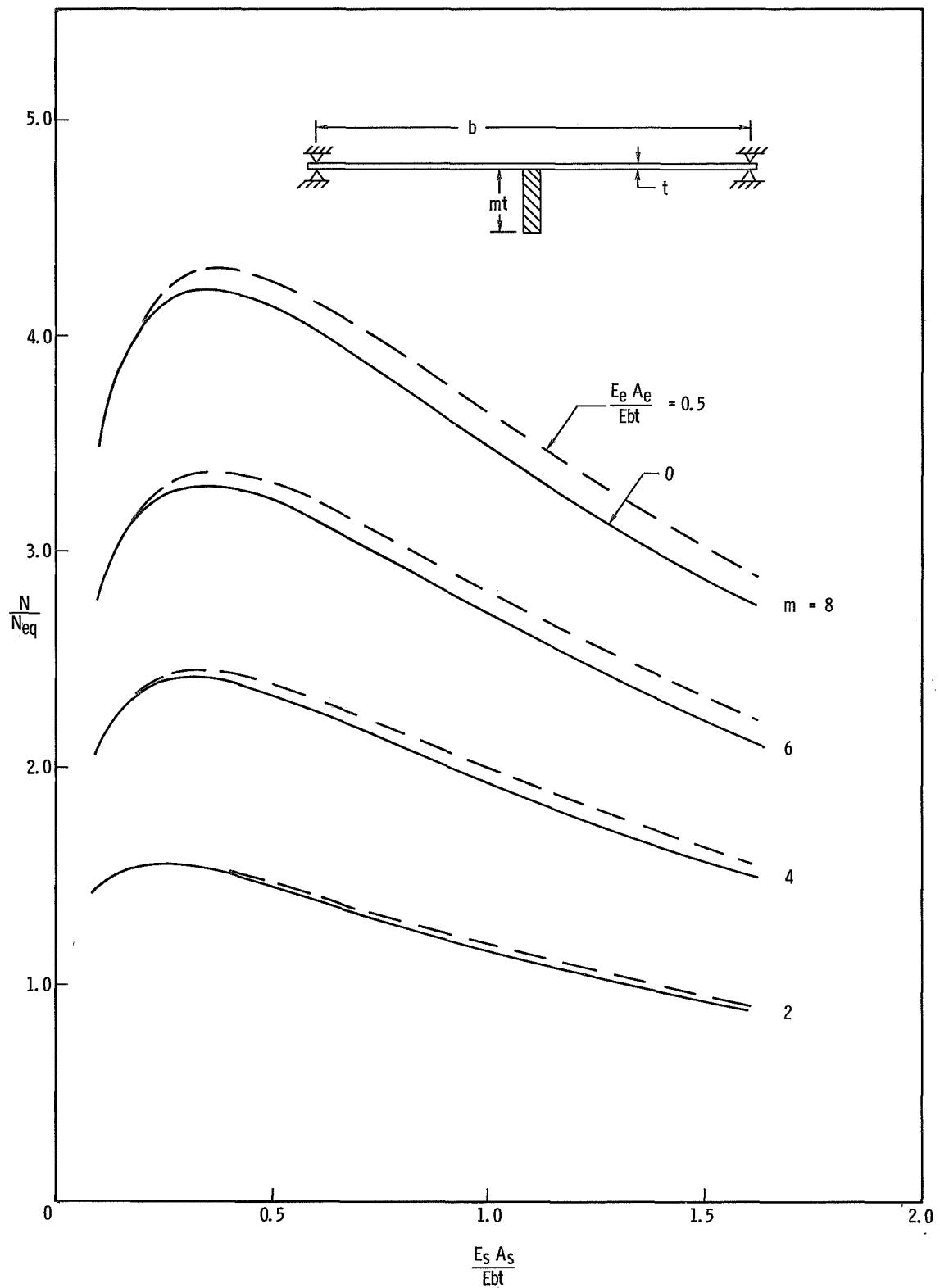


Figure 4.- Symmetric buckling of an aluminum plate stiffened by a central rectangular boron-epoxy stiffener.

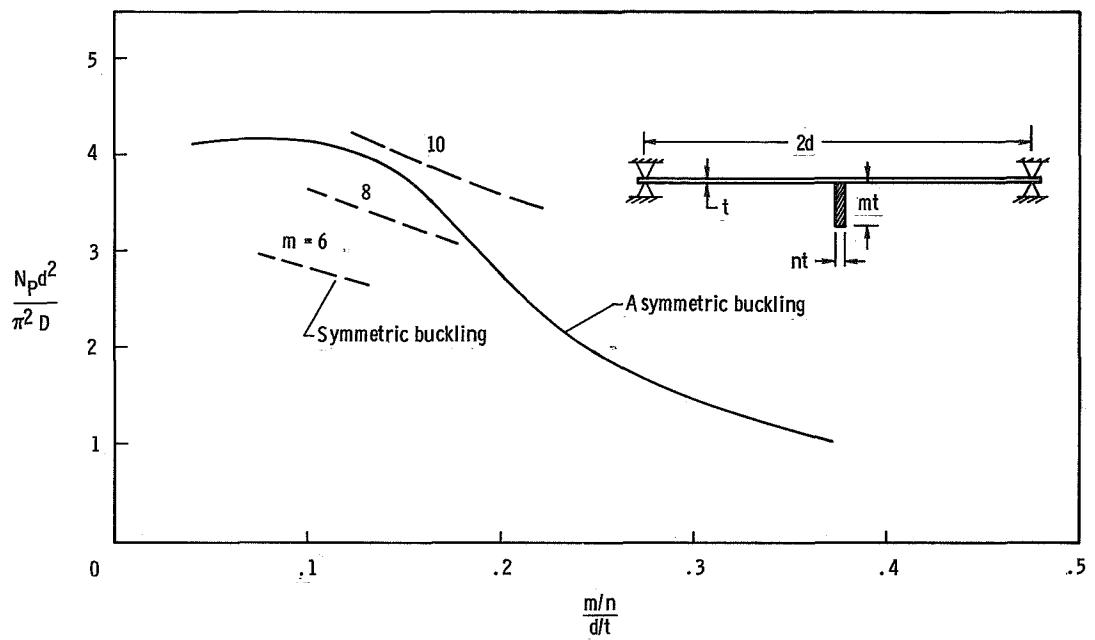


Figure 5.- Local buckling of an aluminum plate reinforced with a central boron-epoxy stiffener ( $n = 2.0$ ).

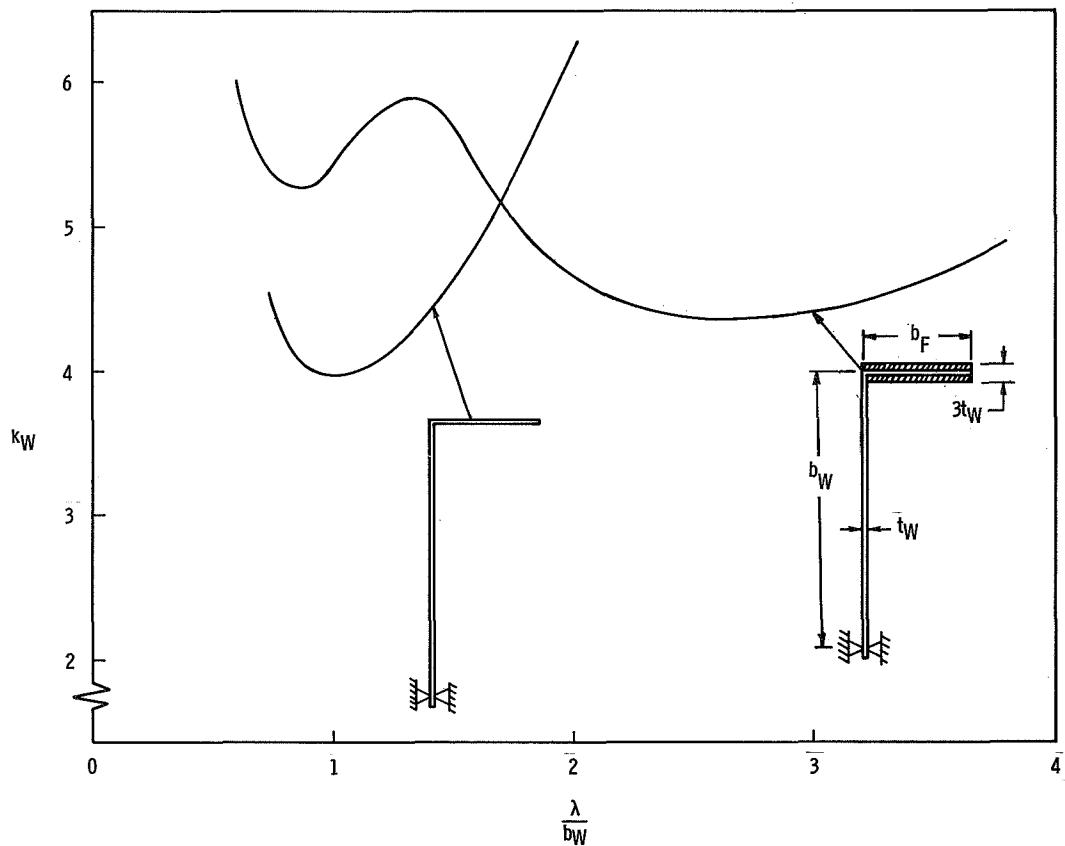


Figure 6.- Local buckling of a Z-stiffener with and without boron reinforcement on the outstanding flange.  $\frac{b_F}{b_W} = 0.40$ .

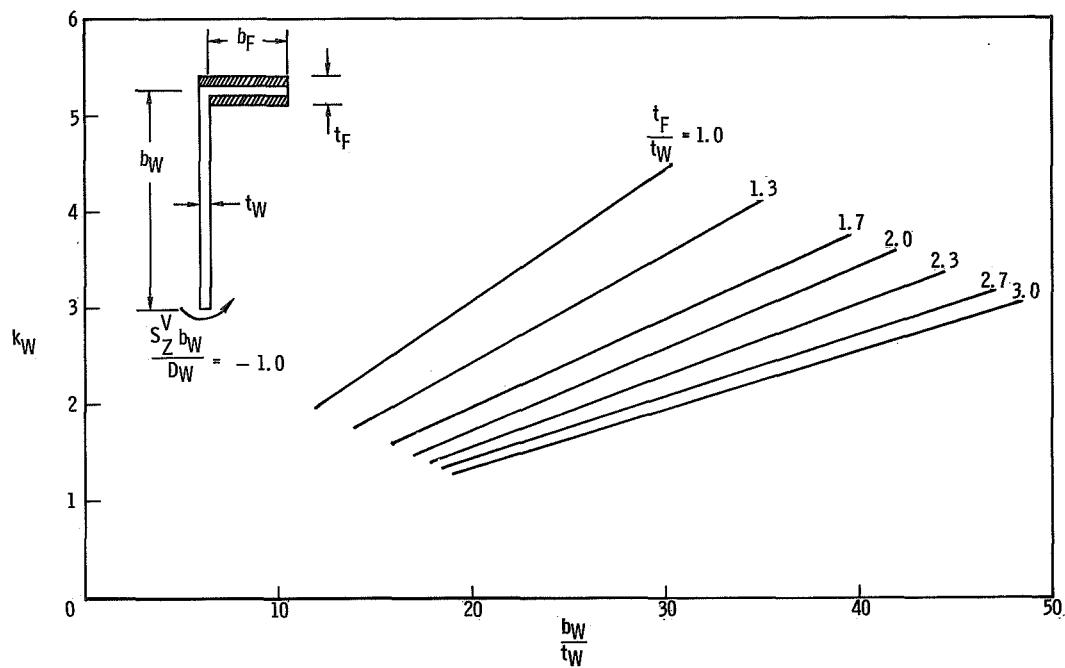


Figure 7.- Buckling of stiffener with reinforced flange in beam-column mode.  $\frac{b_F}{b_W} = 0.40$ .

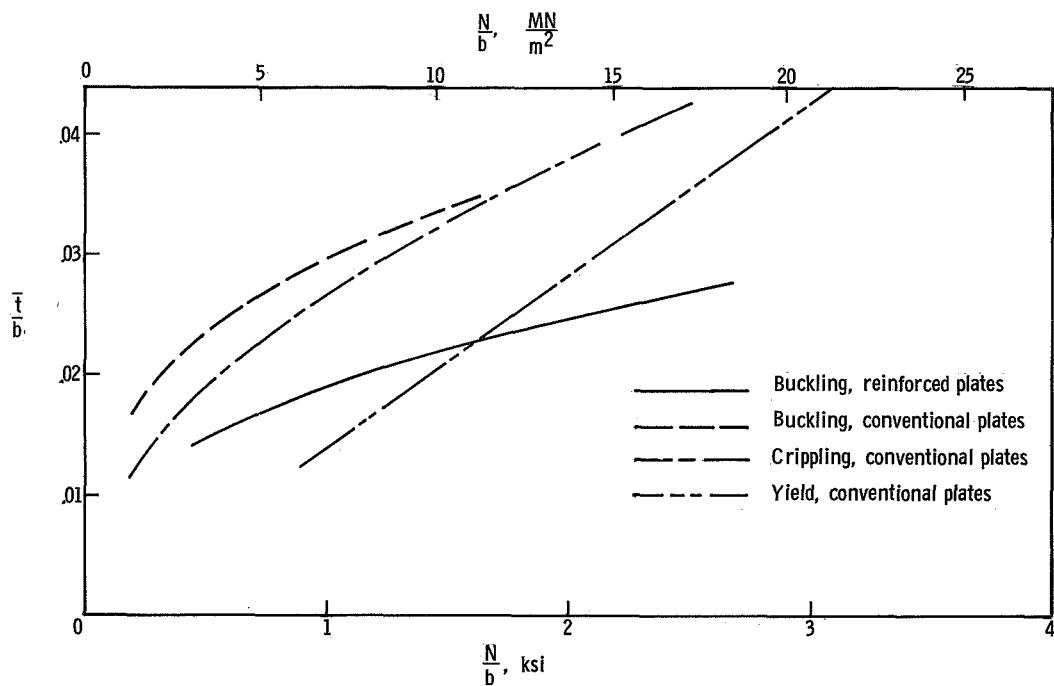


Figure 8.- Comparison of the efficiency of reinforced plates and plates without reinforcement. The curve for reinforced plates is for plates reinforced by a central composite stiffener with  $m = 8$  and  $n = 2$ , the most efficient stiffener found.

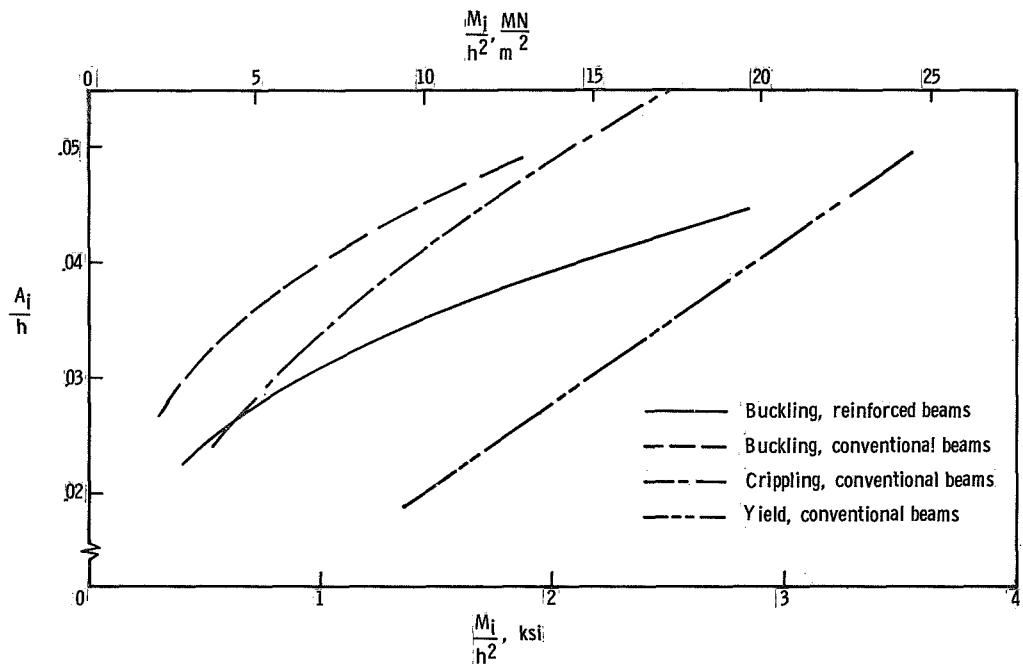


Figure 9.- Comparison of efficiency of multiweb beams with reinforced cover skins and beams without the reinforcement. The curve for reinforced beams is for beams with the cover reinforced by central composite stiffeners with  $m = 8$  and  $n = 2$ , the most efficient stiffeners found.

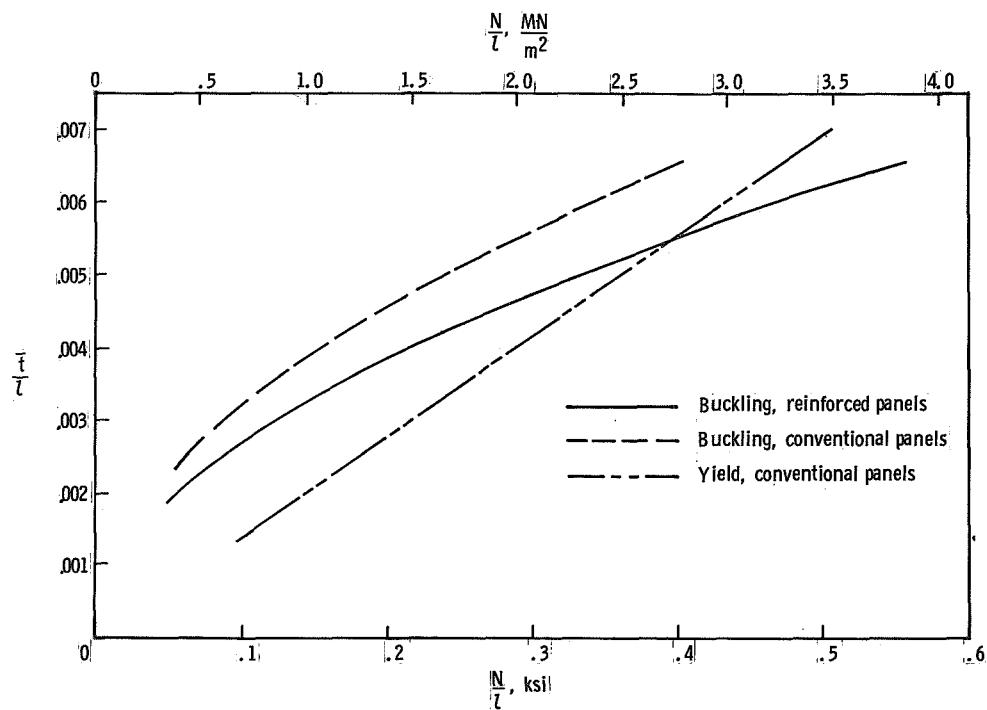


Figure 10.- Comparison of the efficiency of Z-stiffened panels with skin reinforcement and panels without the reinforcement. The curve for reinforced panels is for panels with cover skins reinforced by central composite stiffeners with  $m = 8$  and  $n = 2$ , the most efficient stiffeners found.

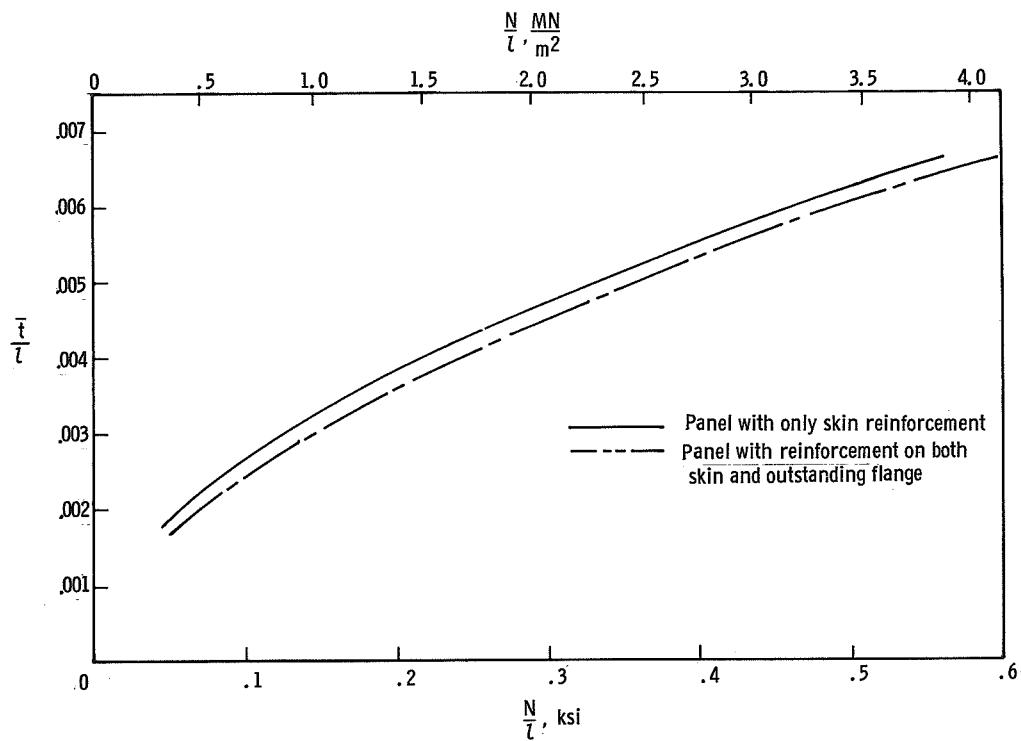


Figure 11.- Comparison of the efficiency of Z-stiffened panels with reinforcement only on the cover skin and panels with reinforcement on both the cover skin and the outstanding flange of the Z-stiffener.

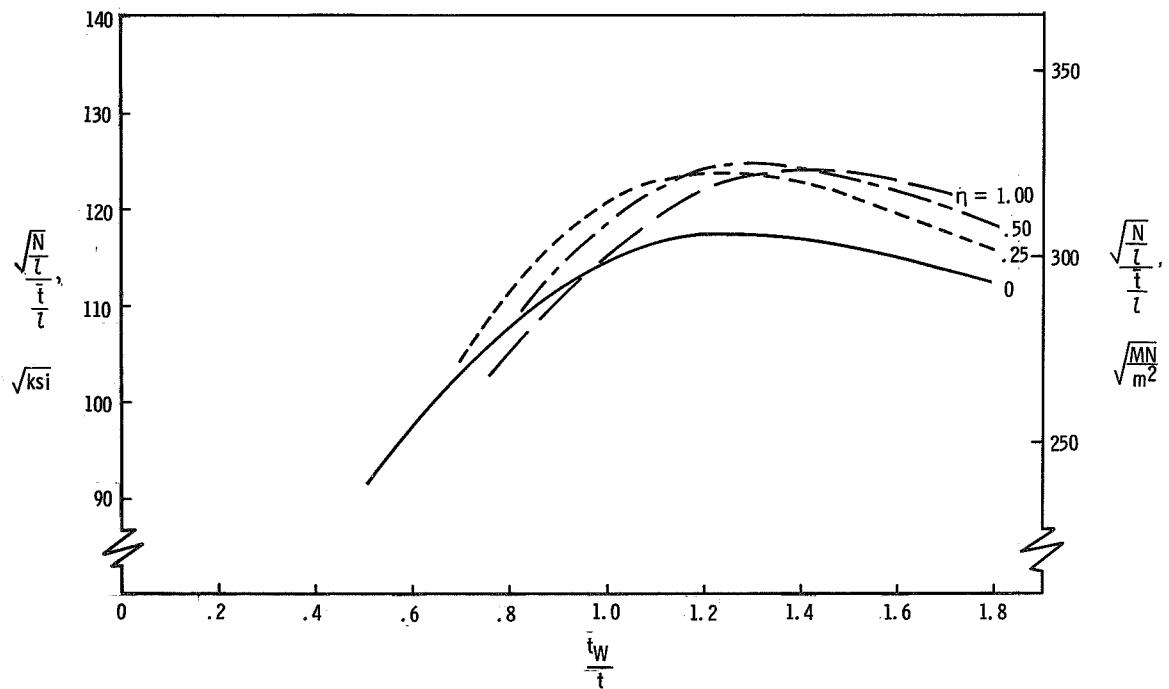


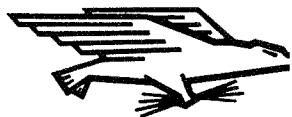
Figure 12.- Effect of flange reinforcement on structural efficiency of Z-stiffened panels with  $\frac{b}{t} = 55$ .

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